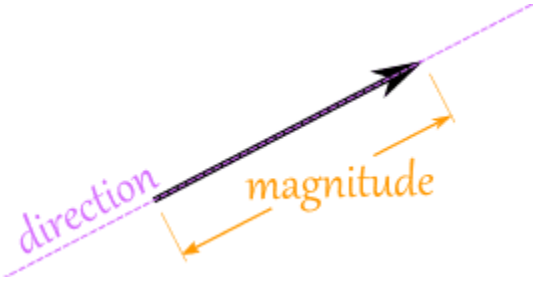
Vectors

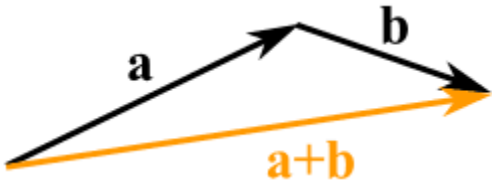
This is a vector:

A vector has **magnitude** (how long it is) and **direction**:

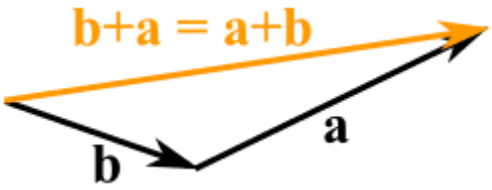


The length of the line shows its magnitude and the arrowhead points in the direction.

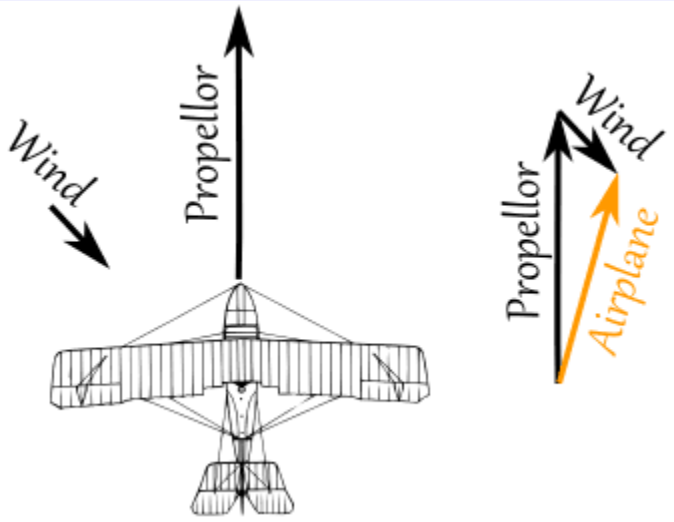
You can add two vectors by simply joining them head-to-tail:



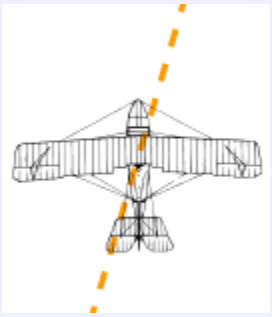
And it doesn't matter which order you add them, you get the same result:



Example: A plane is flying along, pointing North, but there is a wind coming from the North-West.



The two vectors (the velocity caused by the propeller, and the velocity of the wind) result in a slightly slower ground speed heading a little East of North.
If you watched the plane from the ground it would seem to be slipping sideways a little.

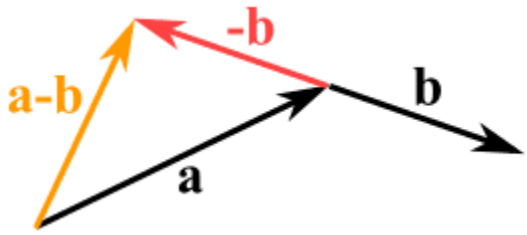


Have you ever seen that happen? Maybe you have seen birds struggling against a strong wind that seem to fly sideways. Vectors help explain that.

Subtracting

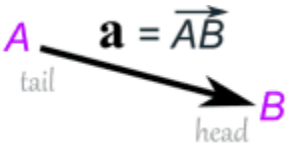
You can also subtract one vector from another:

- first you reverse the direction of the vector you want to subtract,
- then add them as usual:



Other Notation

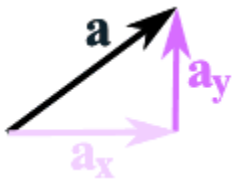
A vector can also be written as the letters of its head and tail with an arrow above, like this:



Calculations

Now ... how do we do the calculations?

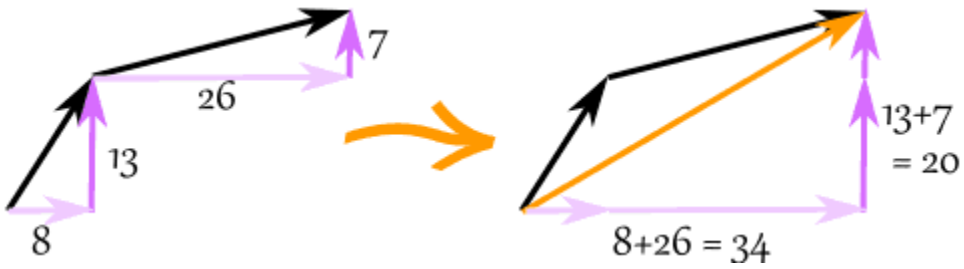
The most common way is to break up a vector into x and y pieces, like this:



The vector **a** is broken up into the two vectors $\mathbf{a_x}$ and $\mathbf{a_y}$

Adding Vectors

And here is how to add two vectors after breaking them into x and y parts:



The vector (8,13) and the vector (26,7) add up to the vector (34,20)

Example: add the vectors **a** = (8,13) and **b** = (26,7)
c = **a** + **b**
c = (8,13) + (26,7) = (8+26,13+7) = (34,20)

Subtracting Vectors

Remember: to subtract, first reverse the vector you want to subtract, then add.

> Example: subtract **k** = (4,5) from **v** = (12,2)
> **a** = **v** + -**k**
> **a** = (12,2) + -(4,5) = (12,2) + (-4,-5) = (12-4,2-5) = (8,-3)

Magnitude of a Vector

The magnitude of a vector is shown by two vertical bars on either side of the vector:

$$|\mathbf{a}|$$

OR it can be written with double vertical bars (so as not to confuse it with absolute value):

$$||\mathbf{a}||$$

You can use Pythagoras' theorem to calculate it:

$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

A vector with magnitude 1 is called a Unit Vector.

Vector vs Scalar

When using vectors we call an ordinary number a "scalar".

Scalar: just a number (like 7 or -0.32) ... definitely not a vector.

A vector is often written in **bold**,

so **c** is a vector, and it has magnitude and direction

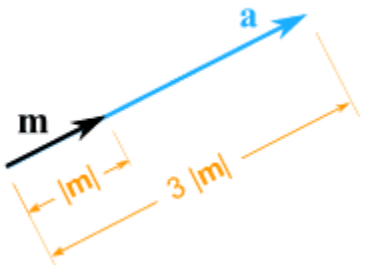
but c is just a value, like 3 or 12.4

> Example: k**b** is actually the scalar k times the vector **b**.

Multiplying a Vector by a Scalar

When you multiply a vector by a scalar it is called "scaling" a vector, because you change how big or small the vector is.

Example: multiply the vector **m** = (7,3) by the scalar 3

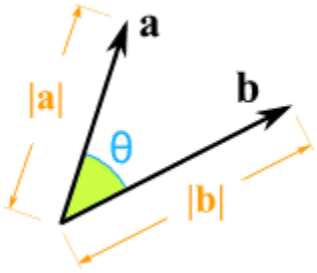


$\mathbf{a} = 3\mathbf{m} = (3\times7, 3\times3) = (21,9)$

It still points in the same direction, but is 3 times longer

(And now you know why numbers are called "scalars", because they "scale" the vector up or down.)

Multiplying a Vector by a Vector (Dot Product and Cross Product)



How do you **multiply two vectors** together? There is more than one way!

- The scalar or Dot Product (the result is a scalar).
- The vector or Cross Product (the result is a vector).