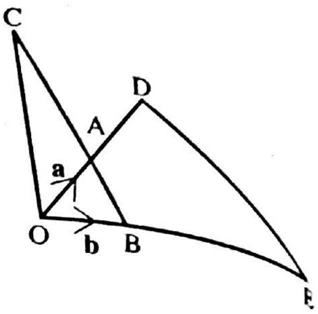
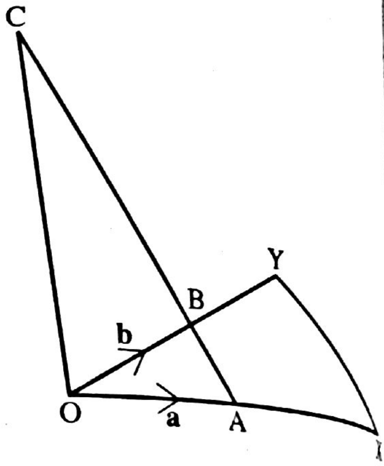
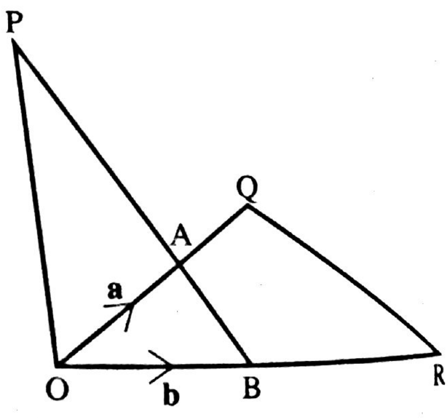
Exercise 10



1. $\overrightarrow{OD} = 2\overrightarrow{OA}$,
   $\overrightarrow{OE} = 3\overrightarrow{OB}$,
   $\overrightarrow{OA} = \mathbf{a}$ and
   $\overrightarrow{OB} = \mathbf{b}$.
   (a) Express $\overrightarrow{OD}$ and $\overrightarrow{OE}$ in terms of **a** and **b** respectively.
   (b) Express $\overrightarrow{BA}$ in terms of **a** and **b**.
   (c) Express $\overrightarrow{ED}$ in terms of **a** and **b**.
   (d) Given that $\overrightarrow{BC} = 4\overrightarrow{BA}$, express $\overrightarrow{OC}$ in terms of **a** and **b**.
   (e) Express $\overrightarrow{EC}$ in terms of **a** and **b**.
   (f) Use the results for $\overrightarrow{ED}$ and $\overrightarrow{EC}$ to show that points E, D and C lie on a straight line.



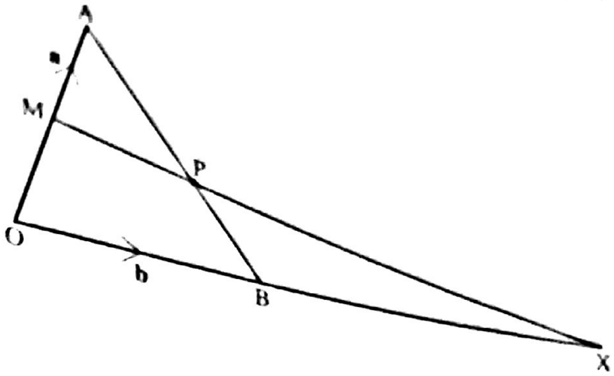
2. $\overrightarrow{OY} = 2\overrightarrow{OB}$,
   $\overrightarrow{OX} = \frac{5}{2}\overrightarrow{OA}$,
   $\overrightarrow{OA} = \mathbf{a}$ and
   $\overrightarrow{OB} = \mathbf{b}$.
   (a) Express $\overrightarrow{OY}$ and $\overrightarrow{OX}$ in terms of **b** and **a** respectively.
   (b) Express $\overrightarrow{AB}$ in terms of **a** and **b**.
   (c) Express $\overrightarrow{XY}$ in terms of **a** and **b**.
   (d) Given that $\overrightarrow{AC} = 6\overrightarrow{AB}$, express $\overrightarrow{OC}$ in terms of **a** and **b**.
   (e) Express $\overrightarrow{XC}$ in terms of **a** and **b**.
   (f) Use the results for $\overrightarrow{XY}$ and $\overrightarrow{XC}$ to show that points X, Y and C lie on a straight line.



3. $\overrightarrow{OA} = \mathbf{a}$,
   $\overrightarrow{OB} = \mathbf{b}$,
   $\overrightarrow{AQ} = \frac{1}{2}\mathbf{a}$,
   $\overrightarrow{BR} = \mathbf{b}$ and
   $\overrightarrow{AP} = 2\overrightarrow{BA}$.
   (a) Express $\overrightarrow{BA}$ and $\overrightarrow{BP}$ in terms of **a** and **b**.
   (b) Express $\overrightarrow{RQ}$ in terms of **a** and **b**.
   (c) Express $\overrightarrow{QA}$ and $\overrightarrow{QP}$ in terms of **a** and **b**.
   (d) Using the vectors for $\overrightarrow{RQ}$ and $\overrightarrow{QP}$, show that R, Q and P lie on a straight line.

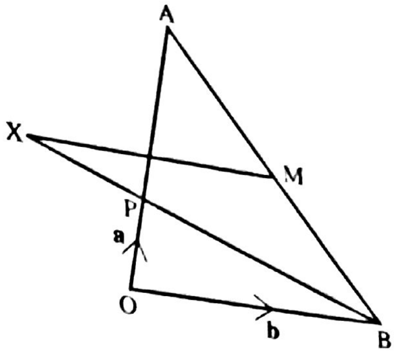
4. In the diagram, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the mid-point of OA and P lies on AB such that $\overrightarrow{AP} = \frac{2}{3}\overrightarrow{AB}$.

(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AP}$ in terms of **a** and **b**.
(b) Express $\overrightarrow{MA}$ and $\overrightarrow{MP}$ in terms of **a** and **b**.
(c) If X lies on OB produced such that OB = BX, express $\overrightarrow{MX}$ in terms of **a** and **b**.
(d) Show that MPX is a straight line.



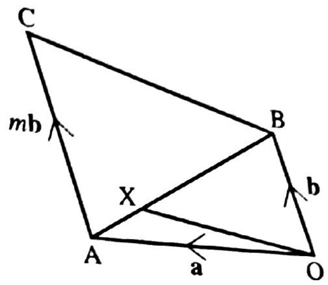
5. $\overrightarrow{OP} = \mathbf{a}$,
$\overrightarrow{OA} = 3\mathbf{a}$,
$\overrightarrow{OB} = \mathbf{b}$ and
M is the mid-point of AB.

(a) Express $\overrightarrow{BP}$ and $\overrightarrow{AB}$ in terms of **a** and **b**.
(b) Express $\overrightarrow{MB}$ in terms of **a** and **b**.
(c) If X lies on BP produced so that $\overrightarrow{BX} = k.\overrightarrow{BP}$, express $\overrightarrow{MX}$ in terms of **a**, **b** and k.
(d) Find the value of k if MX is parallel to BO.



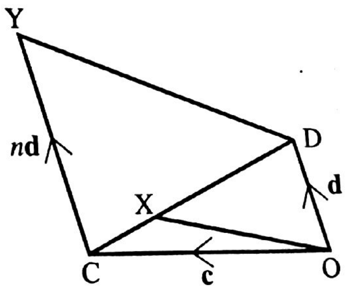
6. AC is parallel to OB,
$\overrightarrow{AX} = \frac{1}{4}\overrightarrow{AB}$,
$\overrightarrow{OA} = \mathbf{a}$,
$\overrightarrow{OB} = \mathbf{b}$ and
$\overrightarrow{AC} = m\mathbf{b}$.

(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**.
(b) Express $\overrightarrow{AX}$ in terms of **a** and **b**.
(c) Express $\overrightarrow{BC}$ in terms of **a**, **b** and m.
(d) Given that OX is parallel to BC, find the value of m.

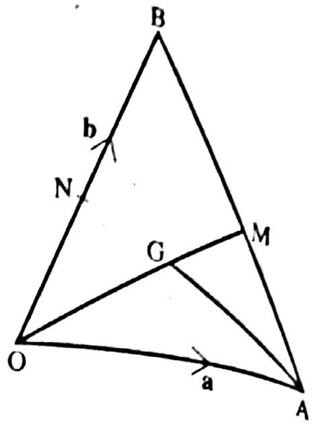


7. CY is parallel to OD,
$\overrightarrow{CX} = \frac{1}{5}\overrightarrow{CD}$,
$\overrightarrow{OC} = \mathbf{c}$,
$\overrightarrow{OD} = \mathbf{d}$ and
$\overrightarrow{CY} = n\mathbf{d}$.

(a) Express $\overrightarrow{CD}$ in terms of **c** and **d**.
(b) Express $\overrightarrow{CX}$ in terms of **c** and **d**.
(c) Express $\overrightarrow{OX}$ in terms of **c** and **d**.
(d) Express $\overrightarrow{DY}$ in terms of **c**, **d** and n.
(e) Given that OX is parallel to DY, find the value of n.

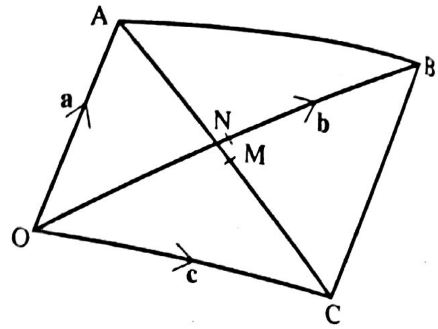


8. M is the mid-point of AB,
N is the mid-point of OB,
$\overrightarrow{OA} = \mathbf{a}$ and
$\overrightarrow{OB} = \mathbf{b}$.



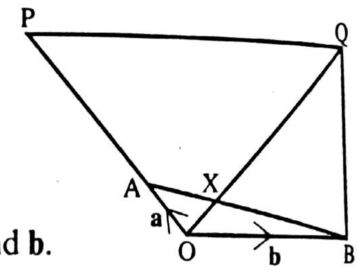
(a) Express $\overrightarrow{AB}$, $\overrightarrow{AM}$ and $\overrightarrow{OM}$ in terms of **a** and **b**.
(b) Given that G lies on OM such that $OG:GM = 2:1$, express $\overrightarrow{OG}$ in terms of **a** and **b**.
(c) Express $\overrightarrow{AG}$ in terms of **a** and **b**.
(d) Express $\overrightarrow{AN}$ in terms of **a** and **b**.
(e) Show that $\overrightarrow{AG} = m\overrightarrow{AN}$ and find the value of m.

9. M is the mid-point of AC and N is the mid-point of OB,
$\overrightarrow{OA} = \mathbf{a}$,
$\overrightarrow{OB} = \mathbf{b}$ and
$\overrightarrow{OC} = \mathbf{c}$.



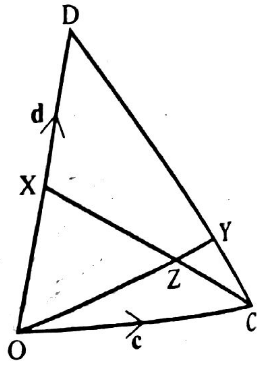
(a) Express $\overrightarrow{AB}$ in terms of **a** and **b**.
(b) Express $\overrightarrow{ON}$ in terms of **b**.
(c) Express $\overrightarrow{AC}$ in terms of **a** and **c**.
(d) Express $\overrightarrow{AM}$ in terms of **a** and **c**.
(e) Express $\overrightarrow{OM}$ in terms of **a** and **c**.
(f) Express $\overrightarrow{NM}$ in terms of **a**, **b** and **c**.
(g) If N and M coincide, write down an equation connecting **a**, **b** and **c**.

10. $\overrightarrow{OA} = \mathbf{a}$ and
$\overrightarrow{OB} = \mathbf{b}$.



(a) Express $\overrightarrow{BA}$ in terms of **a** and **b**.
(b) Given that $\overrightarrow{BX} = m\overrightarrow{BA}$, show that $\overrightarrow{OX} = m\mathbf{a} + (1 - m)\mathbf{b}$.
(c) Given that $OP = 4\mathbf{a}$ and $\overrightarrow{PQ} = 2\mathbf{b}$, express $\overrightarrow{OQ}$ in terms of **a** and **b**.
(d) Given that $\overrightarrow{OX} = n\overrightarrow{OQ}$ use the results for $\overrightarrow{OX}$ and $\overrightarrow{OQ}$ to find the values of m and n.

11. X is the mid-point of OD, Y lies on CD such that
$\overrightarrow{CY} = \frac{1}{4}\overrightarrow{CD}$,
$\overrightarrow{OC} = \mathbf{c}$ and
$\overrightarrow{OD} = \mathbf{d}$.



(a) Express $\overrightarrow{CD}$, $\overrightarrow{CY}$ and $\overrightarrow{OY}$ in terms of **c** and **d**.
(b) Express $\overrightarrow{CX}$ in terms of **c** and **d**.
(c) Given that $\overrightarrow{CZ} = h\overrightarrow{CX}$, express $\overrightarrow{OZ}$ in terms of **c**, **d** and h.
(d) If $\overrightarrow{OZ} = k\overrightarrow{OY}$, form an equation and hence find the values of h and k.