TRANSFORMATION USING MATRICES

A vector could be represented by an ordered pair (x,y) but it could also be represented by a column matrix:

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

Polygons could also be represented in matrix form, we simply place all of the coordinates of the vertices into one matrix. This is called a vertex matrix.

Example

A square has its vertexes in the following coordinates (1,1), (-1,1), (-1,-1) and (1,-1). If we want to create our vertex matrix we plug each ordered pair into each column of a 4 column matrix:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

We can use matrices to translate our figure, if we want to translate the figure x+3 and y+2 we simply add 3 to each x-coordinate and 2 to each y-coordinate.

$$\begin{bmatrix} x_1+3 & x_2+3 & x_3+3 & x_4+3 \\ y_1+2 & y_2+2 & y_2+2 & y_2+2 \end{bmatrix}$$

If we want to dilate a figure we simply multiply each x- and y-coordinate with the scale factor we want to dilate with.

$$3 \cdot \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \end{bmatrix}$$

When we want to create a reflection image we multiply the vertex matrix of our figure with what is called a reflection matrix. The most common reflection matrices are:

for a reflection in the x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

for a reflection in the y-axis

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

for a reflection in the origin

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

for a reflection in the line y=x

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Example

We want to create a reflection of the vector in the x-axis.

$$\vec{A} = \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix}$$

In order to create our reflection we must multiply it with correct reflection matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence the vertex matrix of our reflection is

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ 2 & -2 \end{bmatrix} =$$

$$\begin{bmatrix} (1 \cdot -1) + (0 \cdot 2) & (1 \cdot 3) + (0 \cdot -2) \\ (0 \cdot -1) + (-1 \cdot 2) & (0 \cdot 3) + (-1 \cdot -2) \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -2 & 2 \end{bmatrix}$$

If we want to rotate a figure we operate similar to when we create a reflection. If we want to counterclockwise rotate a figure 90° we multiply the vertex matrix with

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

If we want to counterclockwise rotate a figure 180° we multiply the vertex matrix with

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

If we want to counterclockwise rotate a figure 270°, or clockwise rotate a figure 90°, we multiply the vertex matrix with

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$