Theorems for Congruent Triangles

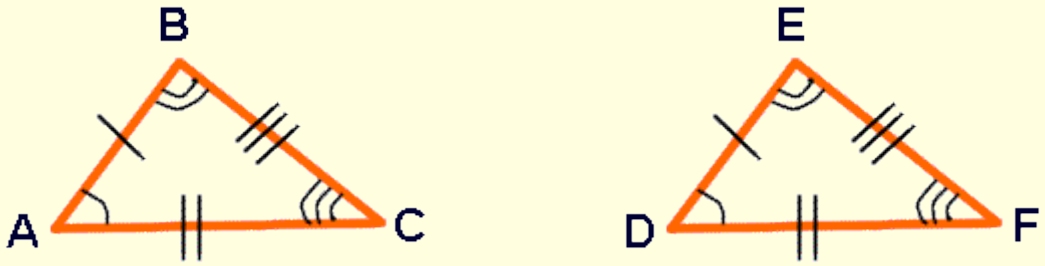
Latest news bulletin:

The most popular congruent figures are triangles!

In many geometrical proofs, it may be necessary to prove that two triangles are congruent to each other. The task may simply be to prove the triangles congruent, or it may be to use these congruent triangles to gain additional information.

When triangles are congruent and one triangle is placed on top of the other, the sides and angles that coincide (are in the same positions) are called **corresponding parts.**

Example:



$$\Delta ABC \cong \Delta DEF$$

When two triangles are congruent, there are 6 facts that are true about the triangles:

- the triangles have 3 sets of congruent (of equal length) sides *and*
- the triangles have 3 sets of congruent (of equal measure) angles.

NOTE: The corresponding congruent sides are marked with small straight line segments called hash marks. The corresponding congruent angles are marked with arcs.

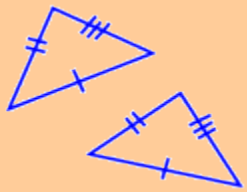
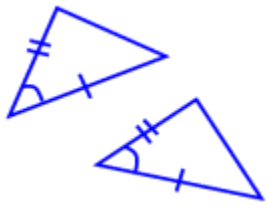
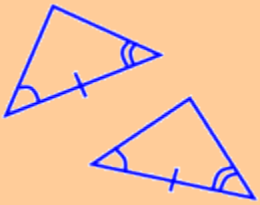
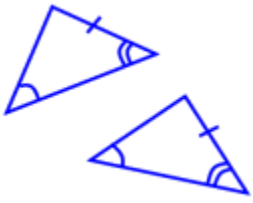
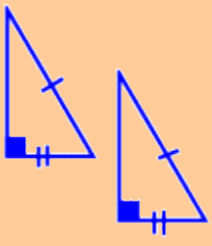
The 6 facts for our congruent triangles example:

$$\overline{AB} \cong \overline{DE} \qquad \measuredangle A \cong \measuredangle D$$
$$\overline{BC} \cong \overline{EF} \qquad \measuredangle B \cong \measuredangle E$$
$$\overline{AC} \cong \overline{DF} \qquad \measuredangle C \cong \measuredangle F$$

Note: The order of the letters in the names of the triangles should display the corresponding relationships. By doing so, even without a picture, you would know that $\measuredangle A$ would be congruent to $\measuredangle D$, and $\overline{BC}$ would be congruent to $\overline{EF}$, because they are in the same position in each triangle name.

Wow! Six facts for every set of congruent triangles!

Fortunately, when we need to PROVE (or show) that triangles are congruent, we do NOT need to show all six facts are true. There are certain combinations of the facts that are sufficient to prove that triangles are congruent. These combinations of facts guarantee that if a triangle can be drawn with this information, it will take on only one shape. Only one unique triangle can be created, thus guaranteeing that triangles created with this method are congruent.

Methods for Proving (Showing) Triangles to be Congruent		
SSS	If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent. (For this method, the sum of the lengths of any two sides must be greater than the length of the third side, to guarantee a triangle exists.)	
SAS	If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. (The included angle is the angle formed by the sides being used.)	
ASA	If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. (The included side is the side between the angles being used. It is the side where the rays of the angles would overlap.)	
AAS	If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. (The non-included side can be either of the two sides that are not between the two angles being used.)	
HL **Right Triangles Only**	If the hypotenuse and leg of one right triangle are congruent to the corresponding parts of another right triangle, the right triangles are congruent. (Either leg of the right triangle may be used as long as the corresponding legs are used.)	

BE CAREFUL!!!
Only the combinations listed above will give congruent triangles.

So, why do other combinations not work?

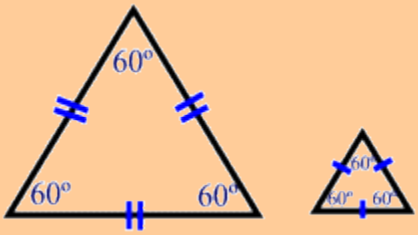
Methods that DO NOT Prove Triangles to be Congruent

AAA

AAA works fine to show that triangles are the same SHAPE (similar), but does **NOT** work to also show they are the same size, thus congruent!

Consider the example at the right.

You can easily draw 2 equilateral triangles that are the same shape but are **not** congruent (the same size).



SSA or ASS

SSA (or ASS) is humorously referred to as the "Donkey Theorem".

This is **NOT** a universal method to prove triangles congruent because it cannot guarantee that one unique triangle will be drawn!!