**The Distributive Law**



Multiplication is distributive over addition: for any numbers *a*, *b* and *c*

*a*(*b* + *c*) = (*ab* + *ac*)

and

(*b* + *c*)*a* = (*ba* + *ca*)

The distributive law allows us to 'multiply out brackets'.



Use pencil and paper to verify the following [identities](http://www.mathstat.strath.ac.uk/basicmaths/glossary.html#identity):

|  |  |
| --- | --- |
| (a) | (*a* + *b*)2 = *a*2 + 2*ab* + *b*2 |
| (b) | (*a* - *b*)2 = *a*2 - 2*ab* + *b*2 |
| (c) | (*a* - *b*)(*a* + *b*) = *a*2 - *b*2 |
| (d) | (*a* + *b*)3 = *a*3 + 3*a*2*b* + 3*ab*2 + *b*3 |
| (e) | (*a* - *b*)3 = *a*3 - 3*a*2*b* + 3*ab*2 - *b*3 |
| (f) | *a*3 - *b*3 = (*a* - *b*)(*a*2 + *ab* + *b*2) |
| (g) | *a*3 + *b*3 = (*a* + *b*)(*a*2 - *ab* + *b*2) |
| (h) | (*a* + *b* + *c*)2 = *a*2 + *b*2 + *c*2 + 2*ab* + 2*bc* + 2*ca* |



Result (c), *a*2 - *b*2 = (*a* - *b*)(*a* + *b*), should be well known to you as the 'difference of two squares formula'. We can illustrate this as follows:

In the diagram below the area of the larger square is *a*2 and the area of the smaller square is *b*2, so that the area of the L-shaped area is *a*2 - *b*2.



If a cut is made along the dotted line and the smaller grey square discarded, the two pieces can be assembled as a rectangle shown in this diagram:



Since the area of a rectangle is length × breadth , the area of the rectangle must equal (*a* - *b*)(*a* + *b*), hence

*a*2 - *b*2 = (*a* - *b*)(*a* + *b*)

**Factorisation**



Factorisation is the process of using the [distributive law](http://www.mathstat.strath.ac.uk/basicmaths/glossary.html#distributivelaw) to reverse the process of multiplying out brackets.

In the expression below, *x* is a **common factor** on the right-hand side and so can be **taken out** to produce the factorised expression on the left-hand side:

3*x* + 5*x* = (3 + 5)*x* = 8*x*

When the value of an expression has to be calculated, the factorised form of the expression is often the easiest to compute.

The first step is always to look for the highest common factor [HCF](http://www.mathstat.strath.ac.uk/basicmaths/glossary.html#hcf) of a set of terms.



To simplify (*a* + 1)(2*a* + 3) - (*a* + 1)(*b* + 3) we note that (*a*+ 1)is a factor of each product.



Sometimes, a common factor is not immediately obvious, but some grouping of terms or rearrangement leads to a factorisation that gives a final expression that is much simpler to evaluate than the initial one.

In the expression *a*2 - 2*pq*- 2*ap*+ *aq* we see that terms 1 and 3 have a common factor as do terms 2 and 4.



To factorise *x*2 + 2*xy*+ *y*2 + *x* + *y* we must notice the [perfect square](http://www.mathstat.strath.ac.uk/basicmaths/glossary.html#perfectsquare).



Do not worry if you feel that you were not able to carry out the last two factorisations on your own: factorisation gets easier with practice!

Factorisations of algebraic expressions are worth looking out for because they usually make evaluation of the expression considerably easier.

For instance, take the second last example and evaluate the expression in your head with *x* = 2 and *y* = 3 first using

*x*2 + 2*xy*+ *y*2 + *x* + *y*

then using

(*x* + *y*)(*x* + *y* + 1)

Which method is easier?

Now do the same using a calculator with *x* = 3.2 and *y* = 1.7

