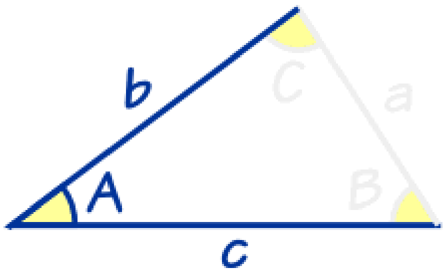
Solving SAS Triangles

"SAS" means "Side, Angle, Side"

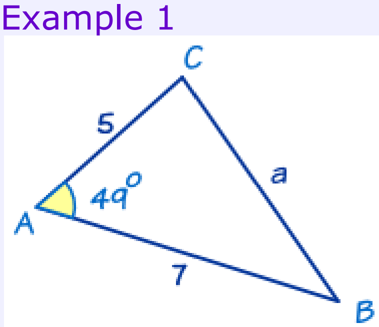


This means we are given two sides and the included angle.

To solve an SAS triangle

- use The Law of Cosines first to calculate the third side of the triangle;
- then use The Law of Sines to find the smaller of the other two angles,
- and finally use the three angles add to 180° to find the last angle.

Example 1



In this triangle we know:

- angle A = 49°
- b = 5
- and c = 7

To solve the triangle we need to find side a and angles B and C.
Because we don't know the angles facing the other two sides we can't use The Law of Sines, so we **must** use The Law of Cosines to find side **a** first:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$a^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos(49°)$

$a^2 = 25 + 49 - 70 \times \cos(49°)$

$a^2 = 74 - 70 \times 0.6560...$

$a^2 = 74 - 45.924... = 28.075...$

$a = \sqrt{28.075...}$

a = 5.298... = **5.30** to 2 decimal places

Now we use the The Law of Sines to find the smaller of the other two angles

Why the smaller angle? Because the inverse sine function gives answers less than 90° even for angles greater than 90°. By choosing the smaller angle we avoid this problem (you can't have two angles greater than 90° in a triangle.) Note: the smaller angle is the one facing the shorter side.

Choose angle B:

sin B / b = sin A / a

sin B / 5 = sin(49°) / 5.298...

Did you notice that we didn't use **a = 5.30**. That number is rounded to 2 decimal places. It's much better to use the unrounded number 5.298... which you should still have on your calculator from the last calculation.

sin B = (sin(49°) × 5) / 5.298... = 0.7122...

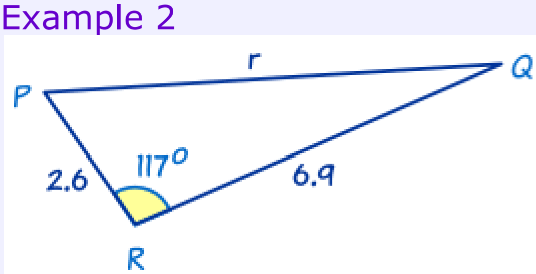
$B = \sin^{-1}(0.7122...)$ **= 45.4°** to one decimal place.

Now we find angle C, which is easy using 'angles of a triangle add to 180°':

C = 180° - 49° - 45.4° = **85.6°** to one decimal place.

Now we have completely solved the triangle i.e. we have found all its angles and sides.

Example 2



This is also an SAS triangle.

First of all we will find r using The Law of Cosines:

$r^2 = p^2 + q^2 - 2pq \cos R$

$r^2 = 6.9^2 + 2.6^2 - 2 \times 6.9 \times 2.6 \times \cos(117°)$

$r^2 = 47.61 + 6.76 - 35.88 \times \cos(117°)$

$r^2 = 54.37 - 35.88 \times (-0.4539...)$

$r^2 = 54.37 + 16.289... = 70.659...$

$r = \sqrt{70.659...}$

r = 8.405... = **8.41** to 2 decimal places

Now The Law of Sines.

Choose the smaller angle? We don't have to! Angle R is greater than 90°, so angles P and Q must be less than 90°.

$\sin P / p = \sin R / r$

$\sin P / 6.9 = \sin(117°) / 8.405...$

$\sin P = (\sin(117°) \times 6.9) / 8.405... = 0.7313...$

$P = \sin^{-1}(0.7313...)$ = **47.0°** to one decimal place.

Now we will find angle Q using 'angles of a triangle add to 180°':

Q = 180° - 117° - 47.0° = **16.0°** to one decimal place