Slopes and Equations of Lines

Let's review our knowledge of slopes
and equations of lines.

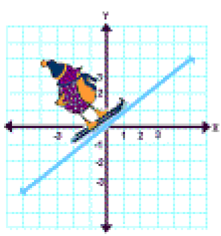
Slopes

The slope of a line is a *rate of change* and is represented by *m*.

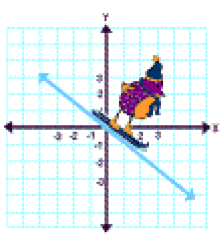
$$Slope = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{Rise}{Run}$$

When a line passes through the points (x_1, y_1) and (x_2, y_2), the slope is

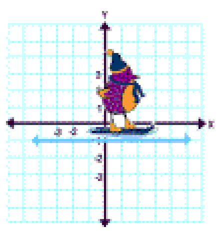
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$



Lines that have a **positive slope**, rise from lower left to upper right. They go up hill.



Lines that have a **negative slope**, decline from upper left to lower right. They go down hill.



Lines that are horizontal have a**slope of zero**. (There is no "rise", creating a zero numerator.)

Equations of lines

Equations of line can take on several forms:

Slope Intercept Form:

[used when you know, or can find, the slope, ***m***, and the*y*-intercept, ***b***.]

$$y = mx + b$$

Point Slope Form:

[used when you know, or can find, a point on the line(x_1, y_1), and the slope, ***m***.]

$$y - y_1 = m(x - x_1)$$

Horizontal Line Form:

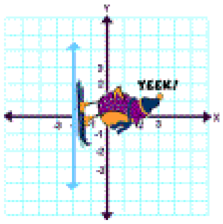
$$y = 3$$ (or any number)

Lines that are horizontal have a slope of zero. They have "run", but no "rise". The rise/run formula for slope always yields zero since rise = 0.

$y = mx + b$
$y = 0x + 3$
$y = 3$

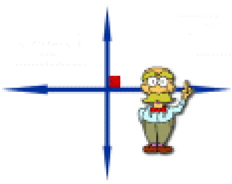
Vertical Line Form:

$$x = -2$$ (or any number)



Lines that are vertical have **no slope (undefined slope).** (There is no "run", creating a zero denominator.)

Lines that are **parallel** have **equal slopes.**



Lines that are **perpendicular** have**negative reciprocal slopes**.
(such as $m = 2$ with $m = -1/2$)

Lines that are vertical have no slope (it does not exist). They have "rise", but no "run". The rise/run formula for slope always has a zero denominator and is undefined.

Examples:

1. Find the slope and *y*-intercept for the equation $3y = -9x + 15$.

First solve for "$y =$": $y = -3x + 5$
Use the form: $y = mx + b$
Answer: the slope (m) is -3
the y-intercept (b) is 5

2. Find the equation of the line whose slope is 4 and crosses the *y*-axis at (0,2).

In this problem $m = 4$ and $b = 2$.
Use the form: $y = mx + b$
Substitute: $\mathbf{y = 4x + 2}$

3. Given that the slope of a line is -3 and the line passes through the point (-2,4), write the equation of the line.

The slope: $m = -3$
The point $(x_1, y_1) = (-2,4)$
Use the form: $y - y_1 = m(x - x_1)$
$y - 4 = -3(x - (-2))$
$\mathbf{y - 4 = -3(x + 2)}$ **ANS.**
If asked to express the answer in "$y =$" form:
$y - 4 = -3x - 6$
$\mathbf{y = -3x - 2}$

4. Find the slope of the line that passes through the points (-3,5) and (-5,-8).

First find the slope: $$m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$m = \frac{5-(-8)}{-3-(-5)} = \frac{13}{2} = 6.5$$

Using either point: (-3,5)
Remember the form: $y - y_1 = m(x - x_1)$
Substitute: $y - 5 = 6.5(x - (-3))$
$\mathbf{y - 5 = 6.5(x + 3)}$ **Ans.**

5. Given that the line is parallel to $y = 4x + 5$ and passes through the point (-2,4), write the equation of the line. Parallel lines have equal slopes, so m = 4. The point $(x_1, y_1) = (-2,4)$ Use the form: $y - y_1 = m(x - x_1)$ $y - 4 = 4(x - (-2))$ $\mathbf{y - 4 = 4(x + 2)}$ **ANS.**	**4. Given $2y = 6x + 12$ and $3y + x = 15$, determine if the lines are parallel, perpendicular, or neither.** Put in "y=" form to observe the slopes. $2y = 6x + 12$ gives $y = 3x + 6$, so $\mathbf{m = 3}$ $3y + x = 15$ gives $y = -1/3\ x + 5$, so $\mathbf{m = -1/3}$ Since the slopes are negative reciprocals, the lines are perpendicular. **ANS.**