Simultaneous Equations - Linear/Quadratic

The simultaneous solution of a linear equation and a quadratic equation is given by the point or points of intersection of the line andparabola representing the equations.

To find the solution, eliminate y (or x) by substitution; and solve the quadratic equation thus formed in x (or y). Then find the corresponding value(s) of y (or x).

Example 19

Solve the following simultaneous equations:

$$y = x + 2$$
$$y = x^2$$

Solution:

$$y = x + 2 \quad \ldots(1)$$
$$y = x^2 \quad \ldots(2)$$

Substitute $y = x^2$ in (1):

$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$\therefore x - 2 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 2 \quad \text{or} \quad x = -1$$

When $x = 2$, $y = 2^2$

$$= 4$$

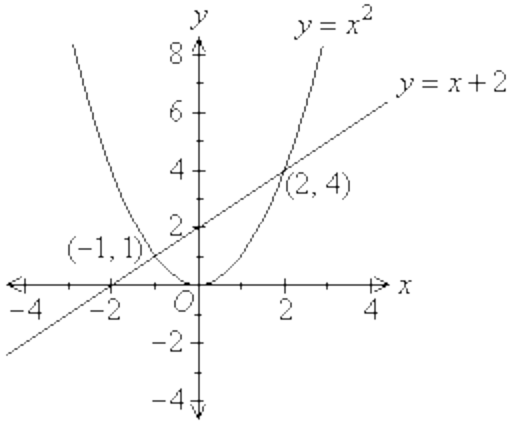
When $x = -1$, $y = (-1)^2$

$$= 1$$

So, the solution set is $\{(-1, 1), (2, 4)\}$.

Note:

The graphical solution can be seen below.



Example 20

Solve the following simultaneous equations:

$y = x^2 - 3x + 4$

$y - x = 1$

Solution:

$y = x^2 - 3x + 4$ ...(1)

$y - x = 1$ ...(2)

Substitute $y = x^2 - 3x + 4$ in (2):

$x^2 - 3x + 4 - x = 1$

$\therefore\ x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$\therefore\ x - 1 = 0$ or $x - 3 = 0$

$x = 1$ or $x = 3$

When $x = 1$, $y - 1 = 1$

$\therefore\ y = 2$

When $x = 3$, $y - 3 = 1$

$\therefore\ y = 4$

So, the solution set is $\{(1, 2), (3, 4)\}$.

Note:

The graphical solution can be seen below.