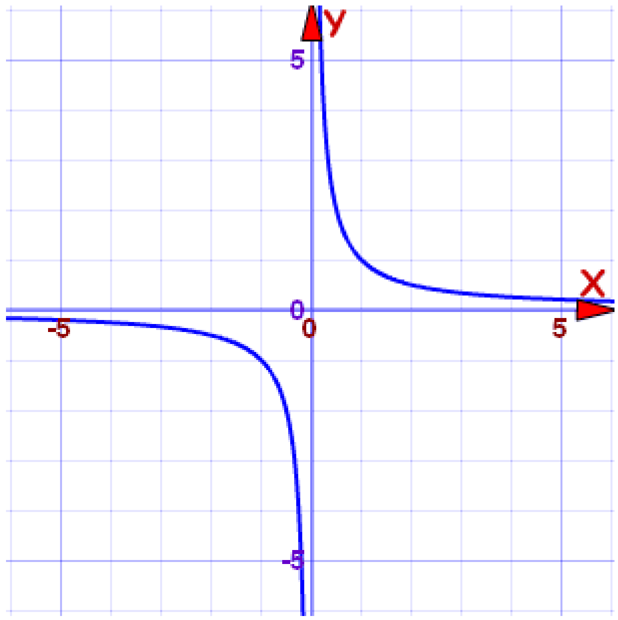
Reciprocal Function

This is the Reciprocal Function:

$$f(x) = 1/x$$

This is its graph:



f(x) = 1/x

It is a Hyperbola.

It is an odd function.

Its Domain is the Real Numbers, **except 0**, because 1/0 is undefined.

Using set-builder notation:

Its Domain is $\{x \in \mathbb{R} \mid x \neq 0\}$

Its Range is also $\{x \in \mathbb{R} \mid x \neq 0\}$

As an Exponent

The Reciprocal Function can also be written as an exponent:

$$f(x) = x^{-1}$$

Even and Odd Functions

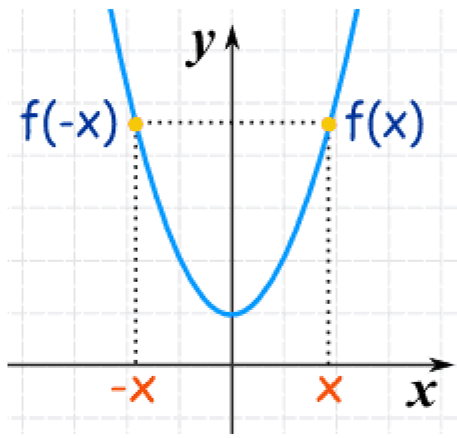
They are special types of functions

Even Functions

A function is "even" when:

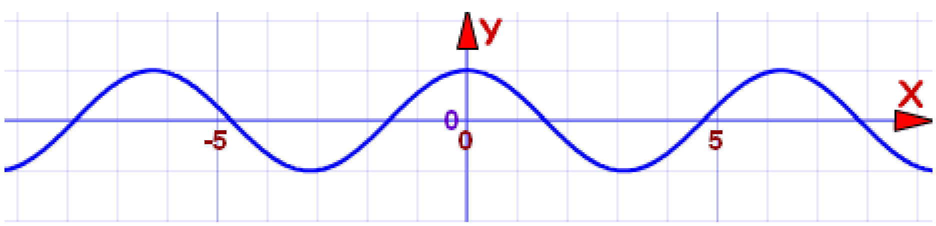
$f(x) = f(-x)$ for all x

In other words there is symmetry about the y-axis (like a reflection):



This is the curve $f(x) = x^2+1$

They got called "even" functions because the functions x^2, x^4, x^6, x^8, etc behave like that, but there are other functions that behave like that too, such as cos(x):



Cosine function: f(x) = cos(x)
It is an even function

But an even exponent does not always make an even function, for example $(x+1)^2$ is **not** an even function.

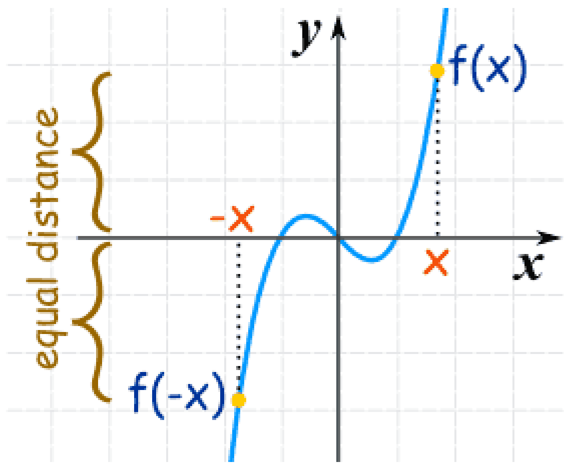
Odd Functions

A function is "odd" when:

$$-f(x) = f(-x) \text{ for all } x$$

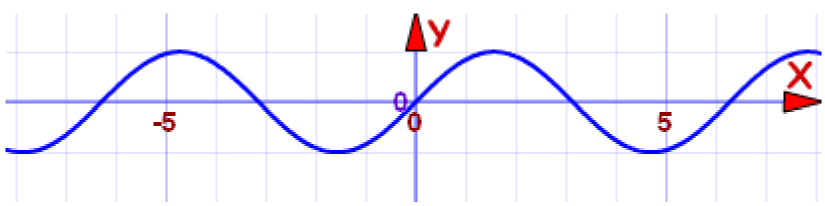
Note the minus in front of f: **-f(x)**.

And we get origin symmetry:



This is the curve $f(x) = x^3 - x$

They got called "odd" because the functions x, x^3, x^5, x^7, etc behave like that, but there are other functions that behave like that, too, such as **sin(x)**:



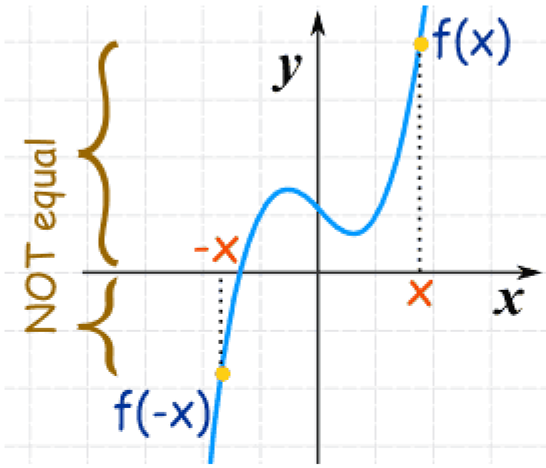
Sine function: f(x) = sin(x)
It is an odd function

But an odd exponent does not always make an odd function, for example x^3+1 is **not** an odd function.

Neither Odd nor Even

Don't be misled by the names "odd" and "even" ... they are just **names** ... and a function does **not have to be**even or odd.

In fact most functions are neither odd nor even. For example, just adding 1 to the curve above gets this:



This is the curve $f(x) = x^3 - x + 1$

It is **not an odd function**, and it is **not an even function** either.
It is neither odd nor even!

Even or Odd?

Example: is $f(x) = x/(x^2-1)$ Even or Odd or neither?

Let's see what happens when we substitute **-x**:

Put in "-x": $f(-x) = (-x)/((-x)^2-1)$

Simplify: $= -x/(x^2-1)$

$= \mathbf{-f(x)}$

So $f(-x) = -f(x)$ and hence it is an **Odd Function**