Pythagoras' Theorem

|  |  |
| --- | --- |
| http://www.mathsisfun.com/images/pythagoras.jpgYears ago, a man named Pythagoras found an amazing fact about triangles: *If the triangle had a right angle (90°) ...**... and you made a square on each of the three sides, then ...**... the biggest square had the****exact same area****as the other two squares put together!* |  |

|  |  |
| --- | --- |
| Pythagoras | It is called "Pythagoras' Theorem" and can be written in one short equation:**a2 + b2 = c2**http://www.mathsisfun.com/geometry/images/pythagoras-squares.gifNote:* **c** is the **longest side** of the triangle
* **a** and **b** are the other two sides
 |

Definition

The longest side of the triangle is called the "hypotenuse", so the formal definition is:

In a right angled triangle:
the square of the hypotenuse is equal to
the sum of the squares of the other two sides.

Sure ... ?

Let's see if it really works using an example.

Example: A ["3,4,5" triangle](http://www.mathsisfun.com/geometry/triangle-3-4-5.html) has a right angle in it.

|  |  |
| --- | --- |
| pythagoras theorem | Let's check if the areas **are** the same:32 + 42 = 52Calculating this becomes:9 + 16 = 25*It works ... like Magic!* |

Why Is This Useful?

If we know the lengths of **two sides** of a right angled triangle, we can find the length of the **third side**. (But remember it only works on right angled triangles!)

How Do I Use it?

Write it down as an equation:

|  |  |  |
| --- | --- | --- |
| abc triangle |   | a2 + b2 = c2 |

Now you can use [algebra](http://www.mathsisfun.com/algebra/index.html) to find any missing value, as in the following examples:

Example: Solve this triangle.

|  |  |
| --- | --- |
| right angled triangle |  a2 + b2 = c252 + 122 = c225 + 144 = c2169 = c2c2 = 169c = √169**c = 13** |

You can also read about [Squares and Square Roots](http://www.mathsisfun.com/square-root.html) to find out why √169 = 13

Example: Solve this triangle.

|  |  |
| --- | --- |
| right angled triangle |  a2 + b2 = c292 + b2 = 15281 + b2 = 225Take 81 from both sides:b2 = 144b = √144**b = 12** |

Example: What is the diagonal distance across a square of size 1?

|  |  |
| --- | --- |
| Unit Square Diagonal |  a2 + b2 = c212 + 12 = c21 + 1 = c22 = c2c2 = 2**c = √2 = 1.4142...** |

It works the other way around, too: when the three sides of a triangle make a2 + b2 = c2, then the triangle is right angled.

Example: Does this triangle have a Right Angle?

|  |  |  |
| --- | --- | --- |
| 10 24 26 triangle |   | Does a2 + b2 = c2 ?* a2 + b2 = 102 + 242 = 100 + 576 =**676**
* c2 = 262 = **676**

They are equal, so ...Yes, it does have a Right Angle! |
|  |  |  |

Example: Does an 8, 15, 16 triangle have a Right Angle?

**Does 8**2 + **15**2 = **16**2?

* 82 + 152 = 64 + 225 = **289**,
* but 162= **256**

So, NO, it does not have a Right Angle

Example: Does this triangle have a Right Angle?

|  |  |  |
| --- | --- | --- |
| Triangle with roots |   | Does a2 + b2 = c2 ?Does (**√**3)2 + (**√**5)2 = (**√**8)2 ?Does 3 + 5 = 8 ?Yes, it does!So this **is** a right-angled triangle |

And You Can Prove The Theorem Yourself !

Get paper pen and scissors, then using the following animation as a guide:

|  |  |
| --- | --- |
|  | * Draw a right angled triangle on the paper, leaving plenty of space.
* Draw a square along the hypotenuse (the longest side)
* Draw the same sized square on the other side of the hypotenuse
* Draw lines as shown on the animation, like this:
* cut sqaure
* Cut out the shapes
* Arrange them so that you can prove that the big square has the same area as the two squares on the other sides
 |

Another, Amazingly Simple, Proof

|  |
| --- |
| Here is one of the oldest proofs that the square on the long side has the same area as the other squares. |
|  |

|  |
| --- |
| Watch the animation, and pay attention when the triangles start sliding around.You may want to watch the animation a few times to understand what is happening.The purple triangle is the important one. |
| before | after |

 |

We also have a [proof by adding up the areas](http://www.mathsisfun.com/geometry/pythagorean-theorem-proof.html).

|  |  |
| --- | --- |
| history | *Historical Note: while we call it Pythagoras' Theorem, it was also known by Indian, Greek, Chinese and Babylonian mathematicians well before he lived !* |