Pythagoras' Theorem

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| http://www.mathsisfun.com/images/pythagoras.jpg  Years ago, a man named Pythagoras found an amazing fact about triangles:    *If the triangle had a right angle (90°) ...*  *... and you made a square on each of the three sides, then ...*  *... the biggest square had the****exact same area****as the other two squares put together!* |  |

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| Pythagoras | It is called "Pythagoras' Theorem" and can be written in one short equation:  **a2 + b2 = c2**  http://www.mathsisfun.com/geometry/images/pythagoras-squares.gif  Note:   * **c** is the **longest side** of the triangle * **a** and **b** are the other two sides |

Definition

The longest side of the triangle is called the "hypotenuse", so the formal definition is:

In a right angled triangle:  
the square of the hypotenuse is equal to  
the sum of the squares of the other two sides.

Sure ... ?

Let's see if it really works using an example.

Example: A ["3,4,5" triangle](http://www.mathsisfun.com/geometry/triangle-3-4-5.html) has a right angle in it.

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| pythagoras theorem | Let's check if the areas **are** the same:  32 + 42 = 52  Calculating this becomes:  9 + 16 = 25  *It works ... like Magic!* |

Why Is This Useful?

If we know the lengths of **two sides** of a right angled triangle, we can find the length of the **third side**. (But remember it only works on right angled triangles!)

How Do I Use it?

Write it down as an equation:

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| abc triangle |  | a2 + b2 = c2 |

Now you can use [algebra](http://www.mathsisfun.com/algebra/index.html) to find any missing value, as in the following examples:

Example: Solve this triangle.

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| right angled triangle | a2 + b2 = c2  52 + 122 = c2  25 + 144 = c2  169 = c2  c2 = 169  c = √169  **c = 13** |

You can also read about [Squares and Square Roots](http://www.mathsisfun.com/square-root.html) to find out why √169 = 13

Example: Solve this triangle.

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| right angled triangle | a2 + b2 = c2  92 + b2 = 152  81 + b2 = 225  Take 81 from both sides:  b2 = 144  b = √144  **b = 12** |

Example: What is the diagonal distance across a square of size 1?

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| Unit Square Diagonal | a2 + b2 = c2  12 + 12 = c2  1 + 1 = c2  2 = c2  c2 = 2  **c = √2 = 1.4142...** |

It works the other way around, too: when the three sides of a triangle make a2 + b2 = c2, then the triangle is right angled.

Example: Does this triangle have a Right Angle?

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| 10 24 26 triangle |  | Does a2 + b2 = c2 ?   * a2 + b2 = 102 + 242 = 100 + 576 =**676** * c2 = 262 = **676**   They are equal, so ...  Yes, it does have a Right Angle! |
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Example: Does an 8, 15, 16 triangle have a Right Angle?

**Does 8**2 + **15**2 = **16**2?

* 82 + 152 = 64 + 225 = **289**,
* but 162= **256**

So, NO, it does not have a Right Angle

Example: Does this triangle have a Right Angle?

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| Triangle with roots |  | Does a2 + b2 = c2 ?  Does (**√**3)2 + (**√**5)2 = (**√**8)2 ?  Does 3 + 5 = 8 ?  Yes, it does!  So this **is** a right-angled triangle |

And You Can Prove The Theorem Yourself !

Get paper pen and scissors, then using the following animation as a guide:

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|  | * Draw a right angled triangle on the paper, leaving plenty of space. * Draw a square along the hypotenuse (the longest side) * Draw the same sized square on the other side of the hypotenuse * Draw lines as shown on the animation, like this: * cut sqaure * Cut out the shapes * Arrange them so that you can prove that the big square has the same area as the two squares on the other sides |

Another, Amazingly Simple, Proof

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| Here is one of the oldest proofs that the square on the long side has the same area as the other squares. | |
|  | |  |  | | --- | --- | | Watch the animation, and pay attention when the triangles start sliding around.  You may want to watch the animation a few times to understand what is happening.  The purple triangle is the important one. | | | before | after | |

We also have a [proof by adding up the areas](http://www.mathsisfun.com/geometry/pythagorean-theorem-proof.html).

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| history | *Historical Note: while we call it Pythagoras' Theorem, it was also known by Indian, Greek, Chinese and Babylonian mathematicians well before he lived !* |