Perfect Squares

In algebra, expressions such as a^2, b^2, a^2b^2 and $(ab)^2$ are called perfect squares. So, $(a+b)^2$ is also a **perfect square** which can be expanded to yield:

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a(a+b)+b(a+b)\\ &= a^2+ab+ba+b^2\\ &= a^2+2ab+b^2 \qquad \{\because ab = ba\}\end{aligned}$$

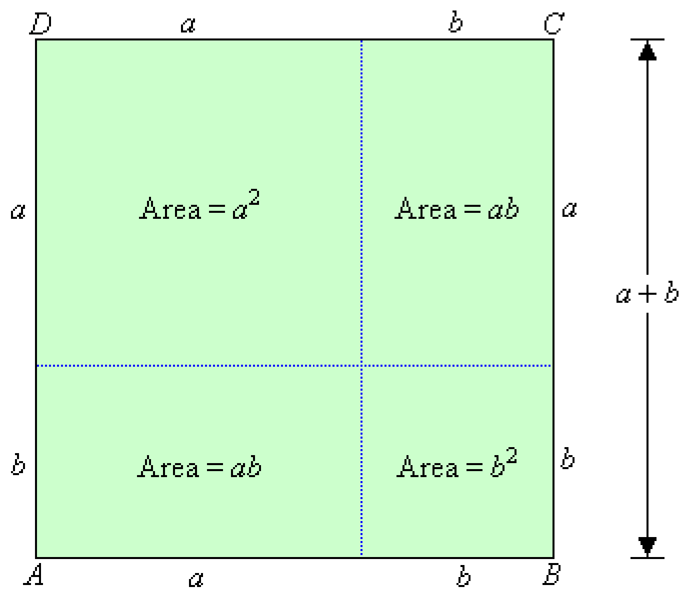
Thus $a^2+2ab+b^2$ is a perfect square since it can be written as the square of $a+b$. That is:

$$(a+b)^2 = a^2+2ab+b^2$$

is a perfect square.

Geometrical Illustration

Consider a square of side $(a+b)$ units as shown in the following diagram.



Area of square $ABCD = (\text{side})^2$

$$= (a+b)^2 \qquad \dots (1)$$

Also, area of square $ABCD$ = sum of areas of four rectangles

$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2 \qquad \dots (2)$$

From (1) and (2), we get:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Likewise, $(a-b)^2$ is a perfect square which can be expanded to yield:

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b)\\ &= a(a-b) - b(a-b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2 \qquad (\because -ab = -ba)\end{aligned}$$

Thus $a^2 - 2ab + b^2$ is a perfect square since it can be written as the square of $a-b$. That is:

$(a-b)^2 = a^2 - 2ab + b^2$ is a perfect square

Applications

The formulas obtained above enable us to expand a perfect square quicker than by using the Distributive Law.

E.g. $(x+5)^2 = x^2 + 2 \times x \times 5 + 5^2$

$= x^2 + 10x + 25$

Often, we write this as follows:

$(x+5)^2 = x^2 + 10x + 25$

Example 9

Expand:

a. $(x+8)^2$

b. $(x-4)^2$

Solution:

a. $(x+8)^2 = x^2 + 16x + 64$ $\{\text{Middle term} = 2 \times x \times 8 = 16x, \text{ last term} = 8^2 = 64\}$

b. $(x-4)^2 = x^2 - 8x + 16$ $\{\text{Middle term} = 2 \times x \times -4 = -8x, \text{ last term} = (-4)^2 = 16\}$