Parallel and Perpendicular Lines

How to use [Algebra](http://www.mathsisfun.com/algebra/index.html) to find [parallel and perpendicular lines](http://www.mathsisfun.com/perpendicular-parallel.html).

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| graph with point (12,5) | Coordinates  We will be using [Cartesian Coordinates](http://www.mathsisfun.com/data/cartesian-coordinates.html), where we mark  a point on a graph by **how far along** and **how far up** it is. |
| Example: The point **(12,5)** is 12 units along, and 5 units up |  |

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| You should also know about [the equation of a line](http://www.mathsisfun.com/equation_of_line.html):  **y = mx + b** |  | Slope-Intercept Form |

Parallel Lines

How do we know when two lines are **parallel**?

Their slopes are the same!

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| Example:  Find the equation of the line that is:   * parallel to **y = 2x + 1** * and passes though the point (5,4) |  | graph |

The slope of **y=2x+1**is: 2

The parallel line must have the same slope!

Let us put that in the ["point-slope" equation of a line](http://www.mathsisfun.com/algebra/line-equation-point-slope.html):

y − y1 = 2(x − x1)

And now put in the point (5,4):

y − 4 = 2(x − 5)

And that is a good answer!

But let's also put it in the ["slope-intercept (**y = mx + b**)"](http://www.mathsisfun.com/equation_of_line.html) form:

y − 4 = 2x − 10

y = 2x − 6

Vertical Lines

But this does not work for vertical lines ... I explain why at the end.

Not The Same Line

Be careful! They may be the same line (just with a different equation), and so would not really be parallel.

How to know if they are really the same line? **Check their y-intercepts** (where they cross the y-axis):

Example: is y = 3x + 2 parallel to y − 2 = 3x ?

For**y = 3x + 2**: the slope is 3, and y-intercept is 2

For **y − 2 = 3x**: the slope is 3, and y-intercept is 2

In fact they are the same line and so are not parallel

Perpendicular Lines

Two lines are Perpendicular if they meet at a right angle (90°).

How do you know if two lines are perpendicular?

When you multiply their slopes, you get -1

This will show you what I mean:

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| graph vertical line |  | These two lines are perpendicular:   |  |  | | --- | --- | | Line | Slope | | y = 2x + 1 | **2** | | y = -0.5x + 4 | **-0.5** |   If we multiply the two slopes we get:  2 × (-0.5) = -1 |

Using It

OK, if we call the two slopes m1 and m2 then we could write:

**m1m2 = −1**

Which could also be:

|  |  |  |
| --- | --- | --- |
| m1 = −1/m2 | *or* | m2 = −1/m1 |

So, to go from a slope to its perpendicular:

* calculate 1/slope (the [reciprocal](http://www.mathsisfun.com/algebra/reciprocal.html))
* and then the negative of that

In other words the **negative of the reciprocal**.

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| Example:  Find the equation of the line that is   * perpendicular to **y = −4x + 10** * and passes though the point **(7,2)** |  | graph |

The slope of **y=−4x+10**is: −4

The **negative reciprocal** of that slope is:

|  |  |  |  |
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| m = − | 1 | = | 1 |
|  |  |
| −4 | 4 |

So the perpendicular line will have a slope of 1/4:

y − y1 = (1/4)(x − x1)

And now put in the point (7,2):

y − 2 = (1/4)(x − 7)

And that is a good answer!

But let's also put it in "y=mx+b" form:

y − 2 = x/4 − 7/4

y = x/4 + 1/4

Vertical Lines

The previous methods work nicely except for one particular case: a **vertical line**:

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| graph vertical line |  | In that case the gradient is undefined (because [you cannot divide by 0](http://www.mathsisfun.com/numbers/dividing-by-zero.html)):   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | m = | |  | | --- | | yA − yB | |  | | xA − xB | | = | |  | | --- | | 4 − 1 | |  | | 2 − 2 | | = | |  | | --- | | 3 | |  | | 0 | | = undefined | |

So just rely on the fact that:

* a vertical line is parallel to another vertical line.
* a vertical line is perpendicular to a horizontal line (and vice versa).