Indices & the Law of Indices

Introduction

Indices are a useful way of more simply expressing large numbers. They also present us with many useful properties for manipulating them using what are called the **Law of Indices**.

What are Indices?

The expression 2^5 is defined as follows:

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

We call "2" the **base** and "5" the **index**.

Law of Indices

To manipulate expressions, we can consider using the Law of Indices. These laws only apply to expressions with the **same base**, for example, 3^4 and 3^2 can be manipulated using the Law of Indices, but we cannot use the Law of Indices to manipulate the expressions 3^5 and 5^7 as their base differs (their bases are 3 and 5, respectively).

Six rules of the Law of Indices

Rule 1: $a^0 = 1$

Any number, except 0, whose index is 0 is always equal to 1, regardless of the value of the base.

An Example:

Simplify 2^0:

$$2^0 = 1$$

Rule 2: $a^{-m} = \frac{1}{a^m}$

An Example:

Simplify 2^{-2}:

$$2^{-2} = \frac{1}{2^2} \qquad \text{Using } a^{-m} = \frac{1}{a^m}$$

$$= \frac{1}{4}$$

Rule 3: $a^m \times a^n = a^{m+n}$

To multiply expressions with the same base, copy the base and add the indices.

An Example:

Simplify 5×5^3: *(note:* $5 = 5^1$*)*

$$5^1 \times 5^3 = 5^{1+3} \qquad \text{Using } a^m \times a^n = a^{m+n}$$

$$= 5^4$$

$$= 5 \times 5 \times 5 \times 5$$

$$= 625$$

Rule 4: $a^m \div a^n = a^{m-n}$

To divide expressions with the same base, copy the base and subtract the indices.

An Example:

Simplify $5(y^9 \div y^5)$:

$$5(y^9 \div y^5) = 5(y^{9-5}) \qquad \text{Using } a^m \div a^n = a^{m-n}$$

$$= 5y^4$$

Rule 5: $(a^m)^n = a^{mn}$

To raise an expression to the nth index, copy the base and multiply the indices.

An Example:

Simplify $(y^2)^6$:

$$(y^2)^6 = y^{2\times 6} \qquad \text{Using } (a^m)^n = a^{mn}$$

$$= y^{12}$$

Rule 6: $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

An Example:

Simplify $125^{2/3}$:

$$125^{2/3} = \left(\sqrt[3]{125}\right)^2 \qquad \text{Using } a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

$$= 5^2 \qquad \text{Recognize cube root of 125 is 5.}$$

$$= 25$$