Graphically Represent the Inverse of a Function

Definition of an Inverse Relation:

If the graph of a function contains a point (*a*, *b*), then the graph of the inverse relation of the function contains the point (*b*, *a*). To graph the inverse of a function, reverse the ordered pairs of the original function.

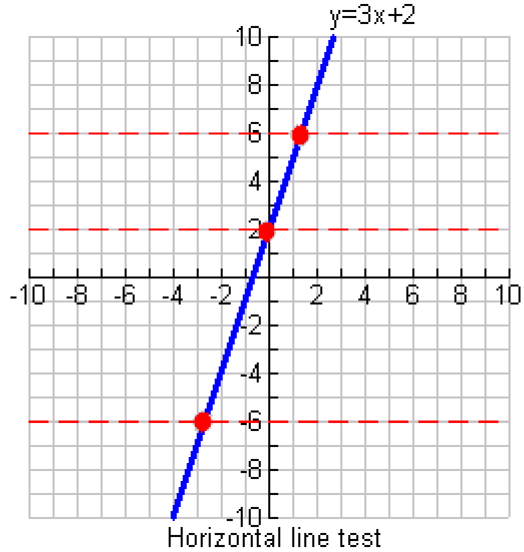
Should the inverse relation of a function $f(x)$ also be a function, thisinverse function is denoted by $f^{-1}(x)$.

"The *x*- and *y*- coordinates switch places!"

Note: The inverse of a function MAY NOT, itself, be a function.

If the inverse of a function is itself also a function, it is referred to as the *inverse function*.

Method 1: Determine graphically if a function has an inverse which is also a function:

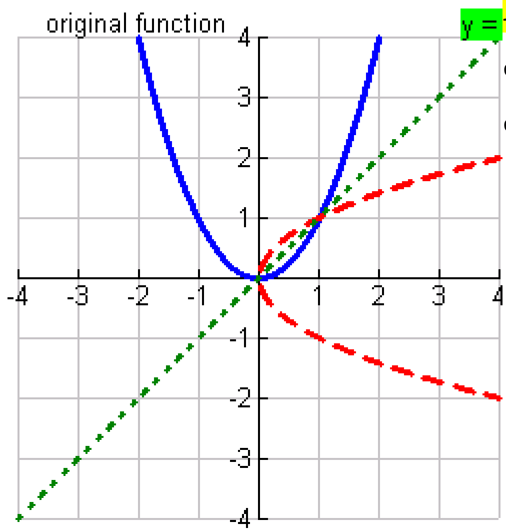


Use the **horizontal line test** to determine if a function has an *inverse function*.

If ANY horizontal line intersects your original function in ONLY ONE location, your function has an inverse which is also a function.

The function $y = 3x + 2$, shown at the right, HAS an *inverse function* because it passes the horizontal line test.

Method 2: Determine graphically if a function has an inverse which is also a function:



If a function has an *inverse function*, the **reflection** of that original function in the **identity line,** $\boldsymbol{y = x}$**,** will also be a function (it will pass the **vertical line test** for functions).

The example at the left shows the original function, $\boldsymbol{y = x^2}$, in **blue**. Its reflection over the identity line $\boldsymbol{y = x}$ is shown in **red** is its inverse relation. The **red** dashed line will not pass the vertical line test for functions, thus $\boldsymbol{y = x^2}$ does **not** have an *inverse function.*
You can see that the inverse relation exists, but it is NOT a function.

NOTE: With functions such as $y = x^2$, it is possible to **restrict the domain** to obtain an *inverse function*for a portion of the graph. This means that you will be looking at only a selected section of the original graph that **will** pass the horizontal line test for the existence of an *inverse function.*

For example:

$y = x^2$ where $x \geq 0$
or
$y = x^2$ where $x \leq 0$

} by restricting the graph in such a manner, you guarantee the existence of an *inverse function* for a portion of the graph. (Other restrictions are also possible.)

The graph of $(f \circ f^{-1})(x)$:

The graph of a function composed with its *inverse function* is the identity line $y = x$.