Commutative, Associative and Distributive Laws

Wow! What a mouthful of words! But the ideas are simple.

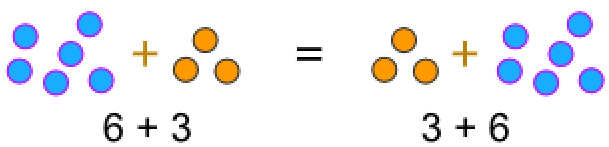
Commutative Laws

The "Commutative Laws" say you can **swap numbers** over and still get the same answer ...

... when you **add**:

$$a + b = b + a$$

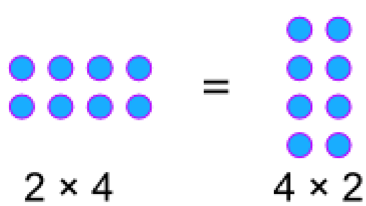
Example:



... or when you **multiply**:

$$a \times b = b \times a$$

Example:

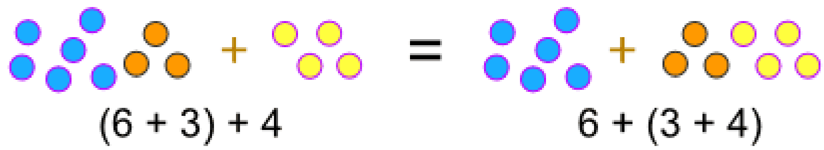


Associative Laws

The "Associative Laws" say that it doesn't matter how you group the numbers (i.e. which you calculate first) ...

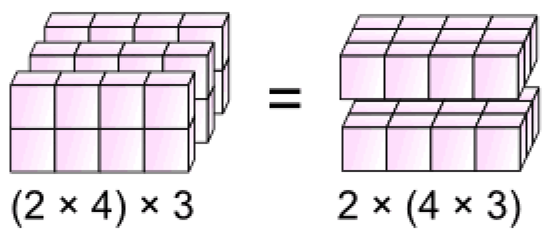
... when you **add**:

$$(a + b) + c = a + (b + c)$$



... or when you **multiply**:

$$(a \times b) \times c = a \times (b \times c)$$



Examples:

This:	$(\mathbf{2 + 4}) + 5 = \mathbf{6} + 5 = \mathbf{11}$
Has the same answer as this:	$2 + (\mathbf{4 + 5}) = 2 + \mathbf{9} = \mathbf{11}$

This:	$(\mathbf{3 \times 4}) \times 5 = \mathbf{12} \times 5 = \mathbf{60}$
Has the same answer as this:	$3 \times (\mathbf{4 \times 5}) = 3 \times \mathbf{20} = \mathbf{60}$

Uses:

Sometimes it is easier to add or multiply in a different order:

What is 19 + 36 + 4?
$19 + 36 + 4 = 19 + \mathbf{(36 + 4)} = 19 + \mathbf{40} = 59$

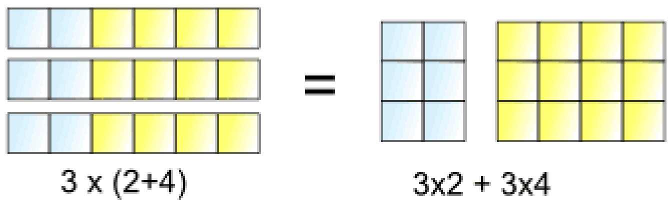
Or to rearrange a little:

What is 2 × 16 × 5?
$2 \times 16 \times 5 = \mathbf{(2 \times 5)} \times 16 = \mathbf{10} \times 16 = 160$

Distributive Law

The "Distributive Law" is the BEST one of all, but needs careful attention.

This is what it lets you do:



3 lots of **(2+4)** is the same as **3 lots of 2** plus **3 lots of 4**

So, the **3×** can be "distributed" across the **2+4**, into **3×2** and **3×4**

And we write it like this:

$$a \times (b + c) = a \times b + a \times c$$

Try the calculations yourself:

- $3 \times (\mathbf{2 + 4}) = 3 \times \mathbf{6} = 18$
- $3\times2 + 3\times4 = 6 + 12 = 18$

Either way gets the same answer.

In English we can say:

You get the same answer when you:

- multiply a number by a **group of numbers added together**, or
- do each **multiply** separately then **add** them

Uses:

Sometimes it is easier to break up a difficult multiplication:

Example: What is 6 × 204 ?

$$6 \times 204 = 6\times200 + 6\times4 = 1{,}200 + 24 = 1{,}224$$

Or to combine:

Example: What is 16 × 6 + 16 × 4?

16 × 6 + 16 × 4 = 16 × **(6+4)** = 16 × **10** = 160

You can use it in subtraction too:

Example: 26×3 - 24×3

26×3 - 24×3 = **(26 - 24)** × 3 = 2 × 3 = 6

You could use it for a long list of additions, too:

Example: 6×7 + 2×7 + 3×7 + 5×7 + 4×7

6×7 + **2**×7 + **3**×7 + **5**×7 + **4**×7 = **(6+2+3+5+4)** × 7 = **20** × 7 = **140**

And those are the Laws!

But Not ...

But don't go too far!

The Commutative Law does **not** work for division:

Example:

- 12 / 3 = **4**, but
- 3 / 12 = **¼**

The Associative Law does **not** work for subtraction:

Example:

(9 – 4) – 3 = 5 – 3 = **2**, but

- 9 – (4 – 3) = 9 – 1 = **8**

The Distributive Law does **not** work for division:

> Example:
> 24 / (4 + 8) = 24 / 12 = **2**, but
> 24 / 4 + 24 / 8 = 6 + 3 = **9**

Summary

Commutative Laws:	$a + b = b + a$ $a \times b = b \times a$
Associative Laws:	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
Distributive Law:	$a \times (b + c) = a \times b + a \times c$