H THE BINOMIAL EXPANSION

Consider $(a+b)^n$. We note that:

- $a+b$ is called a **binomial** as it contains two terms
- any expression of the form $(a+b)^n$ is called a **power of a binomial**
- the **binomial expansion** of $(a+b)^n$ is obtained by writing the expression without brackets.

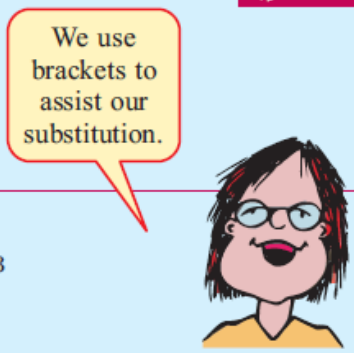
Now $(a+b)^3 = (a+b)^2(a+b)$
$= (a^2+2ab+b^2)(a+b)$
$= a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$
$= a^3+3a^2b+3ab^2+b^3$

So, the **binomial expansion** of $\boxed{(a+b)^3 = a^3+3a^2b+3ab^2+b^3.}$

Example 19 Self Tutor

Expand and simplify using the rule $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$:

a $(x+2)^3$ **b** $(2x-1)^3$



a We substitute $a = x$ and $b = 2$

$\therefore (x+2)^3 = x^3+3\times x^2\times 2+3\times x\times 2^2+2^3$
$= x^3+6x^2+12x+8$

b We substitute $a = (2x)$ and $b = (-1)$

$\therefore (2x-1)^3 = (2x)^3+3\times(2x)^2\times(-1)+3\times(2x)\times(-1)^2+(-1)^3$
$= 8x^3-12x^2+6x-1$

EXERCISE 3H

1 Use the binomial expansion for $(a+b)^3$ to expand and simplify:

a $(x+1)^3$ **b** $(a+3)^3$ **c** $(x+5)^3$

d $(x-1)^3$ **e** $(x-2)^3$ **f** $(x-3)^3$

g $(3+a)^3$ **h** $(3x+2)^3$ **i** $(2x+3y)^3$

2 Copy and complete the argument

$$\begin{aligned}(a+b)^4 &= (a+b)(a+b)^3\\ &= (a+b)(a^3+3a^2b+3ab^2+b^3)\\ &\vdots\end{aligned}$$

3 Use the binomial expansion $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$ to expand and simplify:

a $(x+1)^4$ **b** $(y+2)^4$ **c** $(3+a)^4$ **d** $(b+4)^4$

e $(x-1)^4$ **f** $(y-2)^4$ **g** $(3-a)^4$ **h** $(b-4)^4$

4 Find the binomial expansion of $(a+b)^5$ by considering $(a+b)(a+b)^4$. Hence, write down the binomial expansion for $(a-b)^5$.