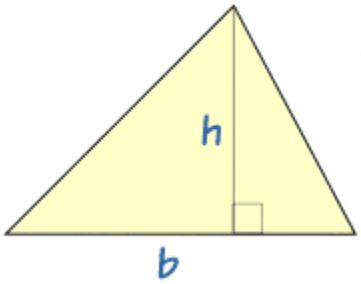
Area of Triangles Without Right Angles

If You Know Base and Height

It is easy to find the area of a right-angled triangle, or any triangle where we are given the base and the height.

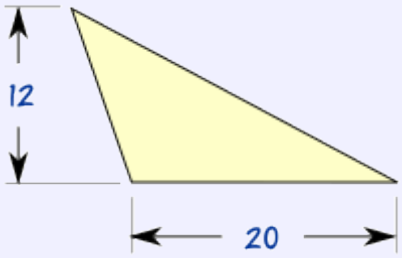


It is simply **half of b times h**

Area = ½bh

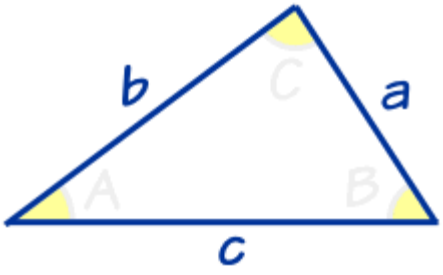
(The Triangles page tells you more about this).

Example: What is the area of this triangle?



Height = h = 12
Base = b = 20
Area = **½ bh** = ½ × 20 × 12 = **120**

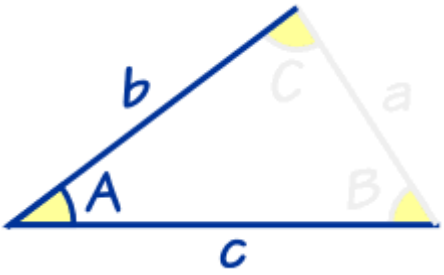
If You Know Three Sides



There's also a formula to find the area of any triangle if we know the lengths of all three of its sides.

This can be found on the Heron's Formula page.

If You Know Two Sides and the Included Angle



If we know two sides and the included angle (SAS), there is another formula (in fact three equivalent formulas) we can use.

Depending on which sides and angles we know, the formula can be written in three ways:

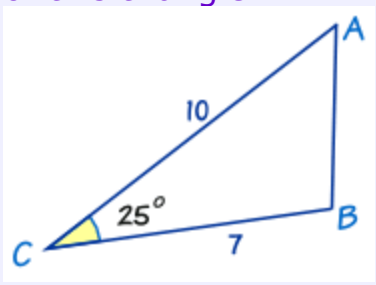
Either **Area = ½ab sin C**

Or **Area = ½bc sin A**

Or **Area = ½ac sin B**

They are really the same formula, just with the sides and angle changed.

Example: Find the area of this triangle:



First of all we must decide what we know.

We know angle C = 25º, and sides a = 7 and b = 10.

So let's get going:

Start with: Area = **½ab sin C**

Put in the values we know: Area = ½ × 7 × 10 × sin(25º)

Do some calculator work: Area = 35 × 0.4226...

Area = **14.8** to one decimal place

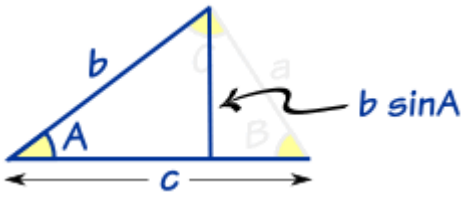
How to Remember

Just think "abc": Area = ½ **a** **b** sin **C**

How Does it Work?

Well, we know that we can find an area if we know a base and height:

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{height}$$



In this triangle:

- the base is: **c**
- the height is: **b × sin A**

Putting that together gets us:

$$\text{Area} = \tfrac{1}{2} \times (c) \times (b \times \sin A)$$

Which is (more simply):

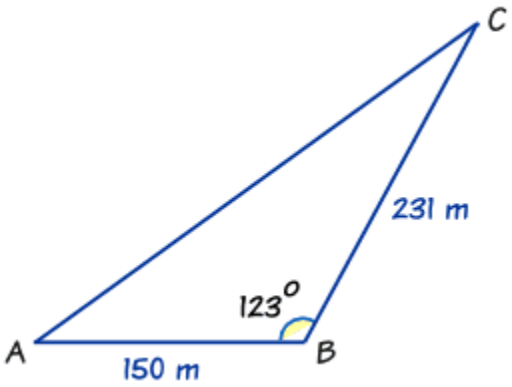
$$\text{Area} = \tfrac{1}{2}bc \sin A$$

By changing the labels on the triangle we can also get:

- Area = ½ab sin C
- Area = ½ca sin B

One more example:

Example: Find How Much Land



Farmer Jones owns a triangular piece of land.
The length of the fence AB is 150 m. The length of the fence BC is 231 m.
The angle between fence AB and fence BC is 123º.
How much land does Farmer Jones own?

First of all we must decide which lengths and angles we know:

- AB = c = 150 m,
- BC = a = 231 m,
- and angle B = 123º

So we use:

$$\text{Area} = \tfrac{1}{2}ca\sin B$$

Start with: Area = **½ca sinB**

Put in the values we know: Area = ½ × 150 × 231 × sin(123º) m^2

Do some calculator work: Area = 17,325 × 0.838... m^2

Area = 14,530 m^2

Farmer Jones has **14,530 m^2** of land