***ALGEBRAIC EXPRESSIONS***

**Variables**

        Letters represent an unknown or generic real number

        Sometimes with restrictions, such as a member of a certain set, or the set of values that makes an equation true.

        Often a letter from the end of the alphabet: *x*, *y*, *z*

        Or a letter that stands for a physical quantity: *d* for distance, *t* for time, etc.

**Constants**

        Fixed values, like 2 or 7.

        Can also be represented by letters: *a*, *b*, *c*, **, *e*, *k*

**Terms**

*Terms* are Separated by + or –



### Factors

*Factors* are multiplied together.

### Coefficients

Coefficients are constant factors that multiply a variable or powers of a variable

The middle term has 2 factors, –3 and *x.* We say that the coefficient of *x* is –3.



The first term has three factors, 2 and two factors of *x.* We say that 2 is the coefficient of *x*2.



The last term is a factor all by itself (although the number 4 could be factored into 2 × 2).



## *SIMPLIFYING ALGEBRAIC EXPRESSIONS*

By “simplifying” an algebraic expression, we mean writing it in the most compact or efficient manner, without changing the value of the expression. This mainly involves *collecting like terms*, which means that we add together anything that can be added together. The rule here is that only *like* terms can be added together.

### Like (or similar) terms

Like terms are those terms which contain the same powers of same variables. They can have different coefficients, but that is the only difference.

**Examples:**

3*x*, *x*, and –2*x* are like terms.

2*x*2, –5*x*2, and are like terms.

*xy*2, 3*y*2 *x*, and 3*xy*2 are like terms.

*xy*2 and *x*2 *y* are **NOT**like terms, because the same variable is not raised to the same power.

### Combining Like terms

Combining like terms is permitted because of the distributive law. For example,

3*x*2 + 5*x*2 = (3 + 5)*x*2 = 8*x*2

What happened here is that the distributive law was used in reverse—we “undistributed” a common factor of *x*2 from each term. The way to think about this operation is that if you have three *x*-squared, and then you get five more *x*-squared, you will then have eight *x-*squared.

**Example:**  *x*2 + 2*x* + 3*x*2 + 2 + 4*x* + 7

Starting with the highest power of *x*, we see that there are four *x*-squareds in all (1*x*2 + 3*x*2). Then we collect the first powers of *x*, and see that there are six of them (2*x* + 4*x*). The only thing left is the constants 2 + 7 = 9. Putting this all together we get

*x*2 + 2*x* + 3*x*2 + 2 + 4*x* + 7

= 4*x*2 + 6*x* + 9

**Parentheses**

        Parentheses must be multiplied out before collecting like terms

You cannot combine things in parentheses (or other grouping symbols) with things outside the parentheses. Think of parentheses as opaque—the stuff inside the parentheses can’t “see” the stuff outside the parentheses. If there is some factor multiplying the parentheses, then the only way to get rid of the parentheses is to multiply using the distributive law.

**Example:**  3*x* + 2(*x* – 4)

= 3*x* + 2*x* – 8

= 5*x* – 8

**Minus Signs: Subtraction and Negatives**

Subtraction can be replaced by adding the opposite

3*x* – 2 = 3*x* + (–2)

#### Negative signs in front of parentheses

A special case is when a minus sign appears in front of parentheses. At first glance, it looks as though there is no factor multiplying the parentheses, and you may be tempted to just remove the parentheses. What you need to remember is that the minus sign indicating subtraction should always be thought of as adding the opposite. This means that you want to add the opposite of the entire thing inside the parentheses, and so you have to change the sign of each term in the parentheses. Another way of looking at it is to imagine an implied factor of one in front of the parentheses. Then the minus sign makes that factor into a negative one, which can be multiplied by the distributive law:

3*x* – (2 – *x*)

= 3*x* + (–1)[2 + (–*x*)]

= 3*x* + (–1)(2) + (–1)(–*x*)

= 3*x* – 2 + *x*

= 4*x* – 2

However, if there is only a plus sign in front of the parentheses, then you can simply erase the parentheses:

3*x* + (2 – *x*)

= 3*x* + 2 – *x*

#### A comment about subtraction and minus signs

Although you can always explicitly replace subtraction with adding the opposite, as in this previous example, it is often tedious and inconvenient to do so. Once you get used to *thinking* that way, it is no longer necessary to actually write it that way. It is helpful to always think of minus signs as being “stuck” to the term directly to their right. That way, as you rearrange terms, collect like terms, and clear parentheses, the “adding the opposite” business will be taken care of because the minus signs will go with whatever was to their right. If what is immediately to the right of a minus sign happens to be a parenthesis, then the minus sign attacks every term inside the parentheses.

***SOLUTIONS OF ALGEBRAIC EQUATIONS***

Up until now, we have just been talking about manipulating algebraic expressions. Now it is time to talk about *equations*. An expression is just a statement like

2*x* + 3

This expression might be equal to any number, depending on the choice of *x*. For example, if *x* = 3 then the value of this expression is 9. But if we are writing an equation, then we are making a statement about its value. We might say

2*x* + 3 = 7

A mathematical equation is either true or false. This equation, 2*x* + 3 = 7, might be true or it might be false; it depends on the value chosen for *x*. We call such equations *conditional*, because their truth depends on choosing the correct value for *x*. If I choose *x* = 3, then the equation is clearly false because 2(3) + 3 = 9, not 7. In fact, it is only true if I choose *x* = 2. Any other value for *x* produces a false equation. We say that *x* = 2 is the *solution* of this equation.

**Solutions**

* The solution of an equation is the value(s) of the variable(s) that make the equation a true statement.

An equation like 2*x* + 3 = 7 is a simple type called a linear equation in one variable. These will always have one solution, no solutions, or an infinite number of solutions. There are other types of equations, however, that can have several solutions. For example, the equation

*x*2 = 9

is satisfied by both 3 and –3, and so it has two solutions.

#### One Solution

This is the normal case, as in our example where the equation 2*x* + 3 = 7 had exactly one solution, namely *x* = 2. The other two cases, no solution and an infinite number of solutions, are the oddball cases that you don’t expect to run into very often. Nevertheless, it is important to know that they can happen in case you do encounter one of these situations.

#### Infinite Number of Solutions

Consider the equation

*x* = *x*

This equation is obviously true for every possible value of *x*. This is, of course, a ridiculously simple example, but it makes the point. Equations that have this property are called *identities*. Some examples of identities would be

2*x* = *x* + *x*

3 = 3

(*x*– 2)(*x* + 2) = *x*2 – 4

All of these equations are true for any value of *x*. The second example, 3 = 3, is interesting because it does not even contain an*x*, so obviously its truthfulness cannot depend on the value of *x*! When you are attempting to solve an equation algebraically and you end up with an obvious identity (like 3 = 3), then you know that the original equation must also be an identity, and therefore it has an infinite number of solutions.

#### No Solutions

Now consider the equation

*x* + 4 = *x* + 3

There is no possible value for *x* that could make this true. If you take a number and add 4 to it, it will never be the same as if you take the same number and add 3 to it. Such an equation is called a*contradiction*, because it cannot ever be true.

If you are attempting to solve such an equation, you will end up with an extremely obvious contradiction such as 1 = 2. This indicates that the original equation is a contradiction, and has no solution.

In summary,

o        An *identity* is always true, no matter what *x* is

o        A *contradiction* is never true for any value of *x*

o        A *conditional equation* is true for some values of *x*